The logical compatibility of Gödel's incompleteness theorem with the physical theory of everything

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Abstract

The goal of this essay is to understand the possible impact of Gödel's incompleteness theorems on the existence of a physical theory of everything from a logical point of view. We will first see what these theorems really means and what should be meant by a theory of everything in Physics. Later in this essay, we will treat the question of a possible formalization of physical theories compatible with Gödel's results. It will become clear that Physics is very different from Mathematics in terms of language construction and proofs. This will allow us to distinguish the idea of completeness between Mathematics and Physics. As a conclusion, we will finally get that Gödel's incompleteness theorem itself does not make the theory of everything logically impossible.

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1 Introduction

In this work, we want to investigate the philosophical consequences of Gödel's incompleteness theorem for physics. Gödel's incompleteness theorem became a historical result in mathematics. This theorem significantly changed the epistemological landscape of the mathematical world since Euclid, who was one of the first mathematicians to formalize mathematics on an axiomatic basis. Gödel's incompleteness theorem is even considered to be one of the most important scientific results of the 20^{th} century. In a few words what Gödel's incompleteness theorem says is the following: if the axioms of the formal system we choose, sufficiently express arithmetic, the Mathematics we will build on it will never be complete and consistent at the same time. More precisely, there exists a sentence in the language of the formal system that must be viewed as true, but which is not provable from the axioms.

Since Euclid, there was in Mathematics the ideal that mathematicians could have drawn all possible mathematical knowledge from a well-chosen axiomatic basis. This was made impossible by Gödel's incompleteness theorem. The ancient dream who motivated a great part of the work done in Mathematics throughout history took an end as a consequence of Gödel's incompleteness theorem. On the other side Physics also has a dream of the same fashion. Indeed the fundamental aim of Physics is to build a theory of everything or in other words a set of laws that describe the whole universe. This physical theory of everything seems to us to be the ideal epistemological horizon toward which Physics tends.

Physics and mathematics are very closely tied together. That is why it seems to us relevant to consider and explore the possible impact of Gödel's incompleteness theorem on Physics from a philosophical point of view. Indeed, since Physics depends so much on Mathematics, we think that Gödel's incompleteness theorem might have some influence for Physics and if it does not, we believe it is not straightforward to show it. More specifically in this work, we want to investigate whether Gödel's incompleteness theorem could imply Physics to be necessarily incomplete as a theory and therefore logically deprived of his ancient dream: the theory of everything. For these reasons the question we choose to tackle this problem with is the following: "Does Gödel's incompleteness theorem imply that the physical theory of everything is impossible?".

To answer this question we decided to organize our work in the following way.[1] First, based on the course "Gödel and recursivity" at EPFL, we will explain what GIT is about, what is its very specific formulation, some basic explanations about its proof, and the kind of logic and mathematics that are involved in it. Second, we will explain thoroughly what we mean by a "physical theory of everything". Third, since GIT is itself based on the idea of "formal system", we will specify in which sense Mathematics is a formal system and which consequences had GIT for the conceptualization of Mathematics as formal system. We will also explain, how we can consider Physics as a formal system. Fourth, we will repeat what completeness means in Mathematics (or at least in which sense it is used in GIT) and we will discuss, what completeness of physics can mean. Finally, putting together all the work done throughout the whole essay, we will conclude, that Gödel's incompleteness theorem does not imply the physical theory of everything to be impossible.

[1] From now on, we will refer to Gödel's incompleteness theorem using the abbreviation GIT, to make the text a little less heavy.

2 The mathematical sense of Gödel's incompleteness theorem

At the beginning of the 20th century, the mathematical community suffered a big crisis called the Fundational crisis. Some essential questions began to emerge such as what are mathematics, is it "true", and the ultimate question : is there a set of primitive truths or axioms from which all the mathematical conjectures can be proven true or false ? Three decades after, the answer to this question has been shown to be negative.

In this first part of our essay, we will introduce the principles and rules that lead to this answer and on which lies modern mathematics. We will do this by first introducing the formalisation of mathematics as it is done nowadays. This will allow us to understand more precisely what the GIT is about and how its proof works. Finally, we will give some philosophical interpretation of the proof and see how it could be interpreted in terms of philosophy of physics and more specifically how it could affect a possible theory of everything.

First, mathematics as it is known since the beginning of the 20th century lie on what we call "formal systems". A formal system is defined by a set of axioms equipped with a set of inference rules. It is important to underline that Gödel's proof applies to ZFC, which is the axiomatic basis behind most theorems in mathematics. The proof in particular shows that every axiomatic basis that contains arithmetic is incomplete.

In a broad sense, arithmetic may be understood as the study of natural numbers combined with all that is possible to achieve with addition, subtraction, multiplication and division. It plays a key role in number theory. We should nonetheless recall that mathematical domains do not have fixed frontiers, as it depends on how the concepts are thought and presented. The frontiers changes from one discovery to another and here appears the need of a mathematical formalisation that could define with more soundness what are the objects we consider. In fact, arithmetic is rather based on a consensus which might get its objectivity in a set of axioms that we call "Robinson arithmetic". Gödel incompleteness theorems precisely work for every set of axioms that is an extension of the so-called "Robinson arithmetic".

Axioms of Robinson arithmetic :	In the standard interpretation : $(x, y \in \mathbf{N})$
$\neg(Sx = 0)$	$x + 1 > 0$
$\neg(Sx = Sy) \vee x = y$	$x + 1 = y + 1 \implies x = y$ (bijection)
$y = 0 \vee \exists x(Sx = y)$	(only 0 does not have a predecessor)
$x + 0 = x$	$x + 0 = x$ (0 is neutral)
$x + Sy = S(x + y)$	$x + (y + 1) = (x + y) + 1$ (associativity)
$x \cdot 0 = 0$	$x \cdot 0 = 0$
$x \cdot Sy = x \cdot y + x$	$x \cdot (y + 1) = x \cdot y + x(distributivity)$

If we wanted to go a step ahead and give an interpretation of those axioms, we could give an interpretation of all the symbols and say that for example, S would be the successor function $S(x) = x + 1$, $\neg$ is the symbol which would exchange the value of truth that we give in our interpretation of a proposition, "=" to be the symbol which identifies two objects, and so on...

Nevertheless, we should keep in mind that we would then be in the semantic process, i.e.

the action of giving a signification to the symbols. We should be careful that axioms are only syntactical symbols that lead to sequences of propositions under the use of some rules. The last proposition would be called the conclusion and the whole set of sequences would be called the proof. In fact, a proof is ultimately a graphical object, a sequence of symbol that graphically lead to a theorem using the rule of inferences. The interpretation is only given a posteriori.

For example, the number 1 in $x + 1$ might be instinctively clear in our interpretation of the natural numbers since it is near our conception of other usual numbers such as 0. Nonetheless, interpreting more complex mathematical objects would require some insight and we would need to inspect how it is constructed syntactically. For example, the standard model is not the only interpretation of the Robinson arithmetic and there exists some valid models containing non-standard integers x that satisfies for example : $\forall n : x > n$. This shows that our definition of numbers should not be limited to our first intuition and we should use solid axioms with the use of syntax. We are in fact searching for fundamental definitions of mathematical objects starting from basic axioms.

Combined with the axioms, we also use inference rules, for example the sequent calculus. A sequent is a set of propositions called the hypothesis and a set of propositions called the conclusions which are joined together by a symbol $\vdash$ called the sequent. We write :

$$A_1, \cdots, A_n \vdash B_1, \cdots, B_m$$

In order to derive more complex conclusions, we introduce formal rules to go from sequents to sequents and this is what we call sequent calculus. The formal rules are also called the inference rules in the sense that they are suppose to infer knowledge from previous sequents. The LK system is a set of twenty rules which formalises first order logic. For example, if a greek letters denotes a set of formulas and A is a formula, an important rule is the cut rules :

$$\frac{\Gamma \vdash \Delta, A \quad \Gamma', A \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}$$

A set of inference rules determines a mathematical formal system. Moreover, all set of inference rules are in fact equivalent in the sense that formal proofs and rules can be translated from one formal system to an other.
The most famous formal system that mathematicians use is called first order logic. It is the one constituted only of variables and predicates. Let just recall that a proposition p is a statement written in the language of the axiomatic system and that can be given a TRUE or FALSE value when interpreted in a given universe that mathematician call a "model". A predicate p(x) is then a proposition that depend on a variable x. For example, when we say "Mars is a planet", "is a planet" is a predicate associated to the variable "Mars" and we say it is TRUE for this variable since we all agree that Mars is indeed a planet in our "universe". First order here, means that the parameters x for the predicates p(x) cannot be functions or predicates, but only variables of fixed values in the domain of the model. A proof of a mathematical statement is then constructed by a sequence of axioms that follows accordingly to the inference rules and leads syntactically to the mathematical statement. We can easily view this as a tree where the theorem proved will be step by step derived until the root of the tree.

Let us now state some important definition encountered in mathematical logic :

- A mathematical formal system is **semantically complete** if for every formula that is true under every valid interpretation of the language and the axioms of the system, it can can be derived from the axioms in a finite number of step using the inference rules : every tautology is a theorem. [2]
- A mathematical formal system is **syntactically complete** if for every statement expressed in the language of the system, either itself or its negation can be derived from the axioms in a finite number of steps using the inference rules. Another way to say it is that no improvable sentence can be added to the system without introducing an inconsistency.
- To the opposite, a mathematical formal system is **syntactically incomplete** if there exists a statement for which itself and its negation cannot be derived from the axioms in a **finite** number of step.

In fact, syntactical completeness is a concept stronger than semantic completeness. In 1929, Gödel proved that first order logic is semantically complete. In 1931, Gödel proved that first order logic is not syntactically complete (as it is shown in GIT).
Note that in the following sections, a syntactically complete theory will be referenced to as a **metasystematical completeness** theory since it is the terminology used in the philosophical domain. Moreover, **empirical completeness** in Philosophy can be interpreted as semantic completeness in mathematics.

With all this being said, we can sketch the proof that Gödel shaped for his famous theorems. It all lies on a **self-reference trick**, which is a usual trick in mathematics (e.g. uncountability of the reals). This might enlight us on the fact that mathematics, the act of representing concepts by symbols, sometimes fails to represent itself or at least some metaconcepts through some representations. For instance : some reals cannot be represented through a fraction which lead to a *tremendous* crisis among the Ancient greeks. In fact, we only need a broader representation to study those irrational numbers (just write $\sqrt{2}$ if one can't write $2,41...$), in the same way that some formulas just can't be represented starting from a given rule of representation (axioms) and need to be accepted as they are to go beyond what is understood to be mathematics. It means, that we would add the formula to the axioms for a larger view of study. This is the same procedure that when looking at our image into a mirror, it might tell us some representations about us, but we would need to find other ways to go further, for example by talking to other people, or **trying to understand the world by doing physics**. By a matter of fact, we only understand our own understanding of the world, but we could believe that if we can understand the world, it might be because it contains resemblances with ourselves ? What GIT tell us, could simply be one possible mathematical application of this universal principle. GIT could even mean, that mathematics by definition would be the art of transcending the limit of representations.

[2] Definition taken from Hunter, Geoffrey, *Metalogic: An Introduction to the Metatheory of Standard First-Order Logic*, University of California Pres, 1971

We can now state the two Gödel incompleteness theorems:

1st theorem of incompleteness: every consistent formal system which axioms extend Robinson arithmetic is incomplete. That is there exists a statement, that can't be proven but should be true if the formal system is consistent.
2nd theorem of incompleteness: every consistent formal system cannot prove its own consistency from its axioms.

We will not explain the proof of the second theorem since it is rather an extension of the first theorem : the improvable statement literally saying that the theory can prove its own consistency. We can however notice that it is this second theorem that definitely put an end to one of the main points of Hilbert program. Let also add, that the axioms that Gödel use for his proof of Robinson arithmetic incompletenes are part of a bigger set of axioms contained in ZFC. ZF was axiomatisation of set theory based on first order logic which was elaborated at the beginning of the 20th century by Ernst Zermelo and Abraham Fraenkel, so to make mathematics more formal and resolve some paradoxes, "C" stands for the axiom of choice added to the ZF-axiomatisation. ZFC is in fact the theory commonly used by mathematicians nowadays.

The reason why the proof of Gödel works for arithmetic is in fact because Gödel made arithmetic talks about itself, by encoding formulas and proofs of arithmetic precisely in arithmetical numbers ! To be more concrete, here $\ulcorner x \urcorner$ means the Gödel number of the symbol of the arithmetical object x.
First, Gödel coded the symbols of the language. For example :

$$\ulcorner \neg \urcorner = 1, \ulcorner \vee \urcorner = 2, \ulcorner 0 \urcorner = 3, \ulcorner = \urcorner = 4, \ulcorner S \urcorner = 5, \ulcorner \forall \urcorner = 6$$

Now the formula "$0 = 0$" is coded using prime numbers to achieve uniqueness (accordingly to the fundamental theorem of arithmetic):

$$\ulcorner 0 = 0 \urcorner = 2^{\ulcorner 0 \urcorner} \cdot 3^{\ulcorner = \urcorner} \cdot 5^{\ulcorner 0 \urcorner}$$

So that each syntactical formula corresponds to a unique natural number. Since a proof is a sequence of formulas linked by inference rules, we code a proof in the same way as we coded formulas as a sequence of symbols. Using this coding, Gödel ingenuously constructed a formula ϕ which basically says :

"there is no proof which proves the formula ϕ".

Mathematically, if $P(x)$ is the formula which states that x is indeed the code of a formula and that there does exist a proof of this formula in Robinson Arithmetic[3], the Gödel formula is written as :

$$\boldsymbol{\phi = \neg(\exists y(y = \ulcorner P(\ulcorner \phi \urcorner) \urcorner))}$$

This sentence is not trivial to construct. Gödel succeeded in making at least one formula speaking about itself by using a diagonal argument, which forces the statement and the code of formulas to meet together for at least one formula : the Gödel sentence. With this sentence involving metamathematics about itself in its own formulation (self referencement), we can deduce :

[3]The sets of formulas and proofs of Robinson arithmetic are decidable so there exists an algorithm which can tell whether a natural number is the code of a formula or a proof. This aspect plays an important role in the detailed proof of GIT.

- If ϕ is false in the standard model of arithmetic, that is the model constructed empirically (for example when interpreting the numbers to find counter-examples), then $\neg\phi$ is true. However $\neg\phi$ syntactically tells that there does exist a proof of ϕ, so in particular ϕ is true and there is a contradiction.

- If ϕ is indeed true in the standard model of arithmetic, then there is no contradiction raised here. Nonetheless, if ϕ also precisely tells that there is no proof for ϕ, we will never be able to verify if ϕ is really true in the standard model of arithmetic.

So finally, can we say that ϕ is true ? In a sense, the answer is positive, if we think truth in a binary way and have faith in our formal system to be consistent (which we always implicitly do otherwise we could do no mathematics and the 2nd GIT namely proves the need of this act of faith), then ϕ cannot be false, so it must be true. But what is remarkable here is that ϕ will never be true on the same level as other formulas could be verified true at the empirical level of the standard model of arithmetic : being derivable from the axioms. This means that truth in mathematics can sometimes given to a statement without any other argument than : "we need it to be true for the axioms we use". Moreover, GIT does not tell how many statements are improvable, but only construct one. So we actually have no idea of the probability that a given conjecture is improvable.

In conclusion, GIT are of philosophical relevance, since they show that mathematics are in fact not reducible in a finite set of primitive "truth", and that the possible results that we can get actually depend on what we first assume, what we dare to consider, and more profoundly what we are able to see. This is in fact what physics is all about.

3 The idea of the physical theory of everything

Since the whole essay is about, whether a physical theory of everything is logically compatible with GIT, in this part we will explain what is meant by a physical theory of everything. Starting by defining briefly what are physical theories. Recall that physical theories are about a primitive ontology on which laws of nature are applied to make predictions from known initial conditions. These terms will be clarified in this part. In this part, we will focus our attention on the theory of everything, since it is the most complete theory physics could have. We will start by discussing what a theory of everything means using one of our main sources, Weinberg's book: *"Dreams of a Final Theory: The Scientist's Search for the Ultimate Laws of Nature"*.

First of all, what is the physical theory of everything. It is a theory that states that the universe is governed by a finite set of fundamental laws, commonly called the laws of nature. These laws are made to work on a primitive ontology which states on what the law is applying. For example, the atom was for a long time the smallest known thing in the universe, so it was the primitive ontology.

Nonetheless, the discovery of quarks is changing this view. The theory of everything says that once one have the fundamental laws and the initial conditions or configuration of the universe, one can explain every phenomena happening in the world and predict all future phenomena. But this makes sense only in a philosophical way. Indeed, in practice, it is impossible to use the laws we have today in particle physics to describe chemistry

for example. And the laws we have are very far from being the laws of a final theory of everything, since there are a lot of phenomena that cannot be explained by the laws we have now. This physical theory of everything is very popular today among the physicists, and it is more a "dream" as said in the title of Weinberg's book since it would be imply to know how the universe works.

It must be said, that the theory of everything is also the "dream" of every modern physicists, since we are very far from the beginning of such a theory. This theory differentiate from other physical theories, because it is the most complete (in a physical point of view) that would exist. Thinking with Weinberg's words: "This theory will bring to an end a certain sort of science, the ancient search for those principles that cannot be explained in terms of deeper principles".[4] It would be a theory containing all the theories we know, and that would describe perfectly every phenomena in the universe, in other words : *the ultimate theory of Physics*. Later in this essay, we will try to know if this theory is compatible with GIT. Now, we would like to go more into details about the nature of this theory, because it is important to understand what we are talking about.

We wanted to introduce a bit of the history of this theory, since it was not always a popular one. Its history starts with the foundation of physics as we know it, with the ancient Greek philosophers. At the time, the notion of a force and law of nature was not clearly formulated and the idea to restrict the world to a finite set of possible phenomena was not very popular. Nonetheless, this was not case of Democritos, a greek pre-Socratic philosopher of the 5th century BC, who stated:

"There is an infinite number of impenetrable atoms, without qualities and indestructible, which move in the void where they are distributed. But when they come close to each other or collide, their aggregation results in water, in fire, in a plant, or in a human being".[5]

This idea lead to the idea of a primitive ontology and to a new domain in Physics: particle physics. Nowadays, this domain is dominant in the research of the theory of everything. This being said, the idea of the laws of nature in the final theory of everything really begins in the 17th century with Descartes, Spinoza and of course Newton. They started to see interactions between objects and to construct theories as Newton did with the gravitation. In his book "Ethics", Spinoza made philosophical assumptions about how "god" or the nature works and he then proved them logically. He used to refer to the laws of nature saying that they were part of "god"[6]. The book has a whole chapter about the fact, that every phenomena can be described by the most little things "bodies" in nature. This can be understood as his primitive ontology of his thinking. Spinoza did not state "atoms" or something else because he wanted a general term that could be better understood. The "bodies" are "moving" or "at rest" which can be interpreted by a physicist as the interactions or laws of nature. Unfortunately, in his book he did not explain and proved his ideas scientifically. His propositions were more about the appreciation of the reader (everyone does not understand the same thing). But, he certainly was one of the firsts to construct a deterministic view of physics and to take the physical theory of everything as possible, true and central (but impossible to know for human beings).

[4] *"Dreams of a Final Theory: The Scientist's Search for the Ultimate Laws of Nature"*, p.18

[5] We took this quote from the slides of the course of Professor Esfeld given the 22nd September 2021.

[6] This can be well understood under the basis of the following quote from Spinoza, *l'Ethique* p.67 : "Par corps, j'entends un mode qui exprime d'une façon définie et déterminée, l'essence de dieu en tant qu'elle est considérée comme chose étendue"

At the same time, Newton built his law of universal gravitation and was the first to really make a theory constituted of laws describing a phenomena, that could be used everywhere without exceptions. This was a revolution for Physics and for science in general, and it led him to write in the preface of his famous book, the **Principia**:

"I wish we could derive the rest of the phenomena of nature [that is, the phenomena not treated in the Principia] by the same kind of reasoning as for mechanical principles. For I am induced by many reasons that they may all depend on certain forces."

This was the beginning of the final theory of everything as we know it. Afterwards, the Electromagnetic force was discovered and explained. And a few times in the last 200 years the physicists thought that they had discovered everything and that the final theory of everything was complete. All of this was before the discovery of the radioactivity in 1896, and all the new discoveries made on atoms in big atoms colliders recently. Since Newton's contribution to physics and science in general, we can say that this theory is supported by a lot of physicists (and unfortunately less philosophers) and that even if it does not directly impact our everyday society there will, at least until we build *the perfect theory*, be research on that subject.

That being said, it is important to understand how the theory of everything is seen by scientists and more specifically if it is accepted within the scientific world. The fact that the theory of everything might be of no use in our everyday life is used a lot by the other types of science as chemistry, medicine or biology to emphasize the importance of the research in their particular subjects. As it is well explained in Weinberg's book[7] We wanted to do a little paragraph to explain why this theory is reductionist and why reductionism does not mean the end of the other types of science as it is well explained in Weinberg's book. First what is reductionism ?

It is the fact that bigger phenomena as chemistry or biology can be entirely explained at a smaller scale by the movement of atoms that can be entirely explained by something smaller until we arrive to the final theory of everything. It is now easy to say that the theory of everything is the definition of a reductionist theory, since it states that we can explain every phenomena using it. But why should it not mean the end of chemistry for example ? Because chemistry is the science of reactions between molecules and even though it would be possible to predict reactions using the theory of everything if it exists, the actual computations would be significantly more complicated and give significantly more information than what we need to describe completely only one reaction. So the answer is yes, the physical theory of everything theory would be reductionist. Nonetheless, it does not mean that it is the only science that matters. Indeed, we need all sciences. We can see science as a giant tree of scientific knowledge where the theory of everything would be the roots and other sciences (as chemistry, biology, geography, etc.) would be the branches or even the leaves of this tree. Naturally, there are connections between the roots and the branches, but the roots do not "explain" directly the branches of the tree. In addition to that, the path to the branches might be impossible to calculate and to measure precisely from the roots, because they would be way too far.

Another important aspect about the theory of everything, that will be very useful for our essay would be the interpretation of the initial condition as a law of nature, or as an axiom of our system. Indeed, how can we turn this theory into a formal system? We know that for a physical law to be applied and to make predictions, we need a perfect

[7] *"Dreams of a Final Theory: The Scientist's Search for the Ultimate Laws of Nature"*, p.51-56

description of the laws of nature but also of the initial conditions. The idea we had was to take the initial conditions of the universe as an axiom and the laws of nature present in the theory as all the other axioms. By doing this, we have a system that would be entirely deterministic, and describe every phenomena would be possible using these axioms. This Idea is also mentioned in Weinberg's book:

"It is possible that eventually the initial conditions will appears as part of the laws of nature"[8]

An example of that would be the steady state cosmology: assuming that the galaxies are rushing away from each other, this theory states that to have an homogeneous universe we would need new matter to be created in the void between galaxies. The steady state cosmology is thus a theory where initial conditions are part of the laws of nature since the creation of the matter is in the theory and has been through the years. The initial conditions would only be an empty space and the disposition of matter through the years would be described by the steady state cosmology, thus the initial conditions needed are parts of the laws for this theory. The theory of everything could be similar to this, the idea to put initial conditions in the axioms of the theory might permit to see the theory of everything as a formal system. Nonetheless, as we will see it later, this does not already mean that GIT can be applied in the context of such a theory.

Last but not least, we also wanted to mention some arguments about the limits of the theory of everything, to better understand its nature. A first argument mentioned in the Weinberg's book:

"We certainly do not have a final theory yet, and we are not likely to discover it soon".[9]

What we wanted to bring in there is the fact that even if this theory exists. We, human beings, are far from discovering it, and we might never be able to formulate this theory of everything. Indeed, probably, our knowledge will never be strong enough to describe this theory. Another argument of Weinberg's book:

"Right now we do not know how to use our standard model of elementary particles to calculate the detailed properties of atomic nuclei, and we are not certain that we will ever know how to do the calculations, even with unlimited computer power at our disposal".[10]

This can also be applied to the theory of everything. Saying that that even if we could formulate this theory theory of everything we would not be able to make predictions using it. In fact, we might need every atoms of the universe only to represent the initial conditions, therefore no simulation of the universe using this theory would be possible. This being said, these arguments are not directly related to the question of our essay, since we want to know if the theory of everything is possible and more specifically if it is logically compatible with GIT. Practical considerations related to our vision of modern science, are not the center of this work, unless they help us to understand what would be the theory of everything in physics. This does not necessarily require high understanding of physics.

The argument we want to bring in throughout this essay is the link between the theory of

[8] *"Dreams of a Final Theory: The Scientist's Search for the Ultimate Laws of Nature"*, p.34
[9] *"Dreams of a Final Theory: The Scientist's Search for the Ultimate Laws of Nature"*, p.6
[10] *"Dreams of a Final Theory: The Scientist's Search for the Ultimate Laws of Nature"*, p.28

everything and GIT. In fact, we know that GIT put an end to the mathematical dream of building mathematics as a closed and complete formal system. For this reason, we are interested to know if the physicist's dream is also impacted by this theorem in some way. This is the question, we will answer later in this essay. Indeed, one possible argument against the logical possibility of the existence of such a theory could be related to GIT. Nevertheless, before discussing this possible argument, we will see how we can understand Physics and mathematics as formal systems in a sense that is meaningful for what are Physics and mathematics as sciences.

4 Mathematics and Physics as formal systems

In this part, we will discuss how we can consider Mathematics and Physics as formal systems and how it is related to GIT. We will see that there are two kinds of formal systems: open formal systems and closed formal systems and that somehow Gödel's incompleteness theorem implies mathematics to be an open system to be consistent with itself. This will help us to understand, how we can or cannot apply GIT to Physics. This part is partly inspired by the article of Cellucci "Godel's incompleteness theorem and the philosophy of open systems".

First of all, it is essential to formulate explicitly, what is meant by "formal system" throughout this essay. We rely on the definition of the Stanford Encyclopedia of Philosophy : a *formal system* is "a system of axioms equipped with rules of inference, which allow one to generate new theorems"[11]. The word "theorem" is used here since this was taken from an article about Mathematics, and the definition is formulated rather from a mathematical perspective. Nevertheless, this can be adapted to the more general context of our essay. We simply consider a "theorem" as a certain knowledge proved using the axioms and the rules of inference given in the formal system.

On one side, an open formal system is a formal system whose own axiomatic basis and rules of inferences can always be changed with respect to new observations. It is a non-self-sufficient formal system, which relies on a constant adjustement with the abstract or concrete reality, it tries to describe.

On the other side, a closed formal system is a formal system based on a closed axiomatic basis from which all the possible knowledge present in the system can be derived and whose rules of inference do not change once they have been established. By "possible knowledge" we mean, the statements that can be formulated within the language of the system. This means that every sentence that can be formulated within the language of the formal system must have a proof derived from the axiomatic basis of this system. The case where the opposite of the sentence is proved to be right, also constitutes a proof. It is the proof of the incorrectness of the original sentence. What is important in the closed formal system is that every possible sentence in the language of the system must be *decidable*. This means that every sentence or its opposite must be provable within the language of the formal system. The closed formal system aims at formalistic self-sufficiency, i.e. at a complete formal independence from other regions of the intellectual

[11]Quoted from the article about Gödel's incompleteness theorem in the Stanford encyclopedia of Philosophy (first paragraph of the "Outline": https://plato.stanford.edu/entries/goedel-incompleteness/ (visited the 1st of May 2022)

world. The closed formal system is also static, since its axiomatic basis and its rules of inference remain the same once they have been established. Closed formal systems are complete and static formal systems.

This formalistic self-sufficiency can also be described as metasystematic completeness. This means that considering the system from outside, everything that can happen in this system must be provable or disprovable within the own language of the system. We will come back in greater lengths to this point in the part dedicated to completeness in Physics versus completeness in Mathematics.

Regarding to what was said in the previous parts, this means that arithmetic and therefore Mathematics itself (since arithmetic is a part of Mathematics) must be an open formal system. Indeed, arithmetic cannot be a closed formal system, which is equivalent to be a complete formal system (as we saw above), because in this case it would be inconsistent and therefore of no use for Mathematics. Arithmetic and thus Mathematics must be open formal systems, i.e. they must be open to adjustements with the abstract reality they aim to describe and explain.

Example given, the axiom of choice was introduced in 1904 by Ernst Zermelo and was used to prove several theorems, which would not have been provable without this axiom. Some of those mathematical results are very controversial such as the "Banach–Tarski paradox", which is a theorem in set-theoretic geometry, which states the following: "Given a solid ball in 3-dimensional space, there exists a decomposition of the ball into a finite number of disjoint subsets, which can then be put back together in a different way to yield two identical copies of the original ball."[12] This axiom was very controversial, but after decades of contestation it was finally accepted and considered to be definitely part of the regular set of axioms of Mathematics. Nowadays, the very majority of the contemporary mathematical society in the world, accepts the axiom of choice.[13]

We see, that Mathematics as a formal system had to include the axiom of choice to evolve. But this must not necessarily be the end of all possible evolution of Mathematics. There are probably many others if not an infinity of possible axioms that are independent with the set of axioms of modern Mathematics. Those axioms could be added to Mathematics in order to describe and prove more of the mathematical world. We use the word "describe", because Mathematics itself already exists as an intellectual reality, and the aim of Mathematics as a science is to developp a language that *describes* or simply formulates this intellectual reality. GIT implies Euclide's and Hilbert's dream of Mathematics as a closed formal system to be impossible.[14] This means that Mathematics must be regarded as a formal system whose axiomatic basis could be changed by some further work (showing the independence of new axioms) and whose rules of inference could be changed or simply improved to fit better the new mathematical realities that might be discovered in

[12]https://brilliant.org/wiki/banach-tarski-paradox/ (visited the 1st of May)

[13]To understand better how this axiom was introduced in modern mathematics and how profound were the mathematical consequences of the acceptation of this axiom, we suggest the reader to refer to the following article :
https://plato.stanford.edu/entries/axiom-choice/#OriChrAxiCho (visited the 1st of May 2022)

[14]Naturally, there are differences between Euclide's and Hilbert's project for Mathematics. Euclide was the first mathematician, who formalized the idea of proof and who tried to derive logically every mathematical statement he was making from his axioms. In this sense, I believe his dream was very closed to Hilbert's dream (and simply to the dream of Mathematics of which it was probably the historical basis). Indeed both wanted to reduce logically the whole mathematical world to one single, complete and perfect axiomatic basis.

the future. As a consequence of GIT, Mathematics must be an open formal system.

This being said about Mathematics, we can come back to the discussion of Physics as a formal system and more specifically of the physical theory of everything as a formal system. In a certain sense, it seems that Physics is a less large science than Mathematics. Indeed, Mathematics is the science of all the sentences that can be expressed within the language of Mathematics, while not all the sentences that can be expressed within the language of Physics (which is partly Mathematics) are the object of Physics as a science. In fact, some of them are just object of studies of Mathematics and others being neither mathematical nor physical are the objects of no science[15]. The objects of studies of Physics are the potential and actual physical events in the universe, and not the sentences that can be created within its own language.

Of course, from a very empirical point of view, the idea of Physics as closed formal system is a non-sense, since Physics always rely on observations of the real world, that can change and evolve. Physics can be regarded as a theoretical structure that we project on the material world in order to explain it scientifically. Physics changes and evolves: under the basis of new observations, new laws might be added to Physics, new interpretations of the already existing physical laws might be introduced to Physics, and Physics is sometimes reorganized from inside (as it was done by Newton and Einstein for example). So therefore Physics as it stands is clearly not a closed formal system, and is rather an open formal system (as we defined it above).

This being said, it is important to understand first what a physical theory is before to continue the discussion about Physics and formal systems. As seen in class during the autumn semester 2021, physical theories are constituted by 3 essential initial components: a primitive ontology, laws of nature and initial conditions. We will give brief definitions of those concepts, since the aim of this essay is not to discuss those concepts very thoroughly, but just to understand how they can be useful for our discussion. First, the primitive ontology is the basic constituents of the matter posited by the theory. This is what makes us experience the world and ourselves. The laws of nature must be regarded as the most basic suppositions, that were confirmed by observations, on which Physics is based. They are not perfect and are only approximations of the perfect laws of nature. The latters, if they exist, would be the most basic truths about the physical world, that organize it and make it exist in a determined way[16]. Last but not least, the initial conditions, are the setting of the system at time t_0, that we fix to be a certain moment in the timeline of the universe. For example, if my system is a pan filled with water, the initial conditions would be the temperature of the water, the size and the form of the pan, the position of the pan in space, the pressure of the air around the pan, etc. All of this would be given at a specific time, that would be considered as t_0. This means, that a physical theory tries to explain the events arising in the system it is observing. It tries to predict how the system in the setting of the initial conditions at time t_0 will behave based on the action of the laws of nature on the primitive ontology, i.e. on the very basic component of the matter of the system.

The physical theory of everything would be composed of the following elements: an exact primitive ontology and a complete set of perfect laws of nature. This would give us the possibility to explain the present state of each system at a time t_1 given its initial

[15]Though they could be the objects of thoughts of Philosophy.

[16]Here "determined" must be considered in a very broad sense. Indeed even probabilistic laws would be "determined" in the sense that they describe the world and organize its existence.

conditions at time t_0. This means that given each system, you can have the possibility to predict perfectly how it will behave until time t_1, though you might not be able do the actual computations (that would be too expensive e.g.). It's the possibility to have the possibility of doing something, but not the direct possibility of doing what you want. With other words, this means that a global understanding is possible, but when it comes to all the actual computational and technical details, the task of understanding becomes almost impossible. Example given, with an infinite energy and infinite amount of time, someone could do all the computations required to deduce how the brain works from the equations of quantum physics, considering those equations a part of the physical theory of everything. But in practice, this would consume so much energy that it would turn out to be impossible.

In addition to that, some computations could be so expensive, that their cost could exceed the total amount of energy contained in the universe, which is finite, and therefore would be impossible in practice. The quantity of energy present in the universe is finite, because the quantity of matter of the universe is finite, this is what is thought in modern Physics. When we speak about physical theory of everything, we mean a physical theory that could explain *in theory* every potential or actual physical event in the universe, where "explain" is meant in a broad sense. Indeed what is meant by an "explanation" has a specific meaning in Physics, which partly differs from what is thought in Mathematics.

Physical explanation does not aim at the possibility of perfect computations of everything, physical explanation is about understanding how the laws of nature work on the universe to make it exists, the way it exists. This means, that physical explanation is not about numerical exactness. Physical explanation is neither about deduction from fundamental logical truths, that are called axioms in Mathematics. Physical explanation is about inductions from empirical observations to fundamental laws. On the other hand, Mathematics is about deducing logically truths from self-evident postulates: the axioms. Mathematics and Physics are indeed very different from each other, and those differences have a significant impact on how Mathematics and Physics are formal systems. Indeed, as seen above Physics as it stands is clearly not a closed formal system. Physics is an open formal system. But what can be said of the physical theory of everything? Provided that a perfect physical theory of everything exists, which kind of formal system would it be? The physical theory of everything in itself would be a fundamental theory, on which all other scientific theories related to natural sciences are related. The theory would be at its fundamental level of explanation of the reality complete. Nonetheless, some phenomena that occur to other more complex levels of reality would not necessarily be explainable directly from this theory.

For example, as it is stated by Chalmers in his book *The conscious mind* the hard problem of consciousness cannot simply be explained by natural sciences and therefore by Physics. Indeed even a complete physical or bio-chemical description of the functioning of the brain would not transmit the very qualitative aspect of what it is like to be conscious. As a second example, as Weinberg says in *Dreams of a final theory* the entropy is a concept important to thermodynamic, that nevertheless disappears at the level of the physics of particles (which would be a good candidate for the physical theory of everything according to him)[17].

For those reasons, as it has already being said in a previous part, the physical theory

[17]In this paraphrase, we refer partly to what Weinberg say at the page 41 of his book.

of everything would be itself complete, but it would not exhaust all scientific researchs in the other fields of natural sciences and not even in other fields of Physics. Natural sciences would naturally act as arrows that point toward the fundamental theory, without being identified with this fundamental theory of everything. On one hand the physical theory of everything would be an open formal system in the sense that the interpretation and the understanding of what goes on to some higher level of reality would remain very open. On the other hand it would be closed, in the sense that the theory with the primitive ontology and the initial conditions would suffice to itself for itself. This final theory would explain what goes on to the most fundamental level of physical reality and it will let open the questions arising from the interpretation of this theory to some higher level of reality.

The question remains how GIT could be used in the context of the theory of everything. Clearly, the setting of the physical theory of everything laws of nature, primitive ontology and initial conditions does not really look like the setting of Mathematics as a formal system, i.e. axioms and rules of inference. The only possible way to apply GIT to the physical theory of everything would be to consider the initial conditions, the laws of nature and the primitive ontology as the axioms of this theory. This process is called "axiomatization" and it was also suggested by Weinberg in *Dreams of a final theory*. The rules of inferences would be then defined, in order to make the whole theory consistent and representative of the physical world. Only then we would approach what Gödel meant by "formal system" in GIT. Nonetheless GIT could only imply Physics and more specifically the physical theory of everything to be metasystematically incomplete, which is not necessarily opposed to completeness as it would be meaningful for Physics. We will see in the next part what this means.

5 Completeness in Physics versus completeness in Mathematics

In this part, we will consider what it would mean for a physical theory to be complete and if there can be a truly complete physical theory. We will compare completeness in Physics with the completeness in Mathematics we discussed previously.

First of all, it is essential to distinguish two types of completeness: empirical completeness and metasystematic completeness. Metasystematic completeness can be said about a discipline formalized as a system and considered from outside. This is the completeness Gödel is talking about in GIT. For example, in Mathematics this means that Mathematics as a system is complete and self-sufficient : every sentence that can be formulated within the language can be proved or disproved within the same system. In other words, metasystematic completeness is about a discipline that is independent, complete and self-sufficient as a system and as a language.

On the other hand empirical completeness, is the completeness about what there is already in the system considered. The question is: "Can my theory truly explain what already exists in the system?". Or thinking with the words of Carlo Cellucci: "Do the rules exhaust the intuitive modes of reasoning actually used in science?".[18] This completeness is not about some invented sentence that cannot be proved or explained within

[18] *Gödel's incompleteness theorem and the philosophy of open systems*, p.108.

the scientific networks of truths[19] that were established in a specific discipline. This completeness is about the sentences that truly make sense within the scientific world of the considered discipline. This completeness is called "empirical" because it is meant to be the completeness of everything that can or could be observed within one scientific world. For Mathematics this would be therefore the completeness of a theory to explain all the different areas of ordinary Mathematics, i.e. already existing and consistent Mathematics. The idea is to observe the mathematical world as it stands and to try to develop one single theory that can explain and prove it all. If this is possible, then we have empirical completeness and if not, we do not.

Regarding Physics, metasystematic completeness would include the metasystematic completeness of one of the languages used in Physics: Mathematics. Nonetheless, this is already metasystematically incomplete because of GIT. Therefore, Physics is metasystematically incomplete. This being said, one could argue, that not all Mathematics is part of Physics, and that therefore the previous argument does not work as easily as it seems. The fact is that even if we consider only the part of Mathematics dedicated to physics, this is very likely to be metasystematically incomplete because of GIT. In addition to that, there are sentences that could be formulated within the language of Physics, but that would not make much sense for the physical world and for Physics itself!

Indeed, Physics is not the science of all the sentences that can be formulated within its own language. Physics is the science that aims at a precise and global understanding of how the material world works. Necessarily, there are some sentences stated in the language of Physics that do not make much sense for Physics itself. For example, sentences about impossible physical events could make absolutely no sense for Physics. A physical theory that cannot prove formally their impossibility from its conceptual basis (i.e. laws of nature, primitive ontology and rules of inference) would be metasystematically incomplete, but not necessarily empirically incomplete. Indeed empirical completeness in Physics is about all the sentences regarding potential and actual physical events. With other words, empirical completeness in Physics is about the sentences that truly make sense for Physics and the existing world surrounding us and not about all the sentences that can be formulated within the language of Physics.

In fact, if an event is impossible, you will not be able to observe it and to say anything about it. Therefore, nothing can be said empirically about this event. For this reason, the inability of showing the impossibility of the event in a physical theory does not imply it to be empirically incomplete. Impossible events are merely ideas and those ideas as they stand are not necessarily part of Physics. They would be probably rather part of Mathematics or Philosophy, or of none of those subjects depending on the context.

To give a very concrete example, let us consider the event "one object or particle exceeding the speed of light" and suppose it is impossible. Then, if our physical theory cannot show the impossibility of this event, this does not mean that our theory is empirically incomplete, but just that it is metasystematically incomplete. Indeed, the very goal of a physical theory is to explain the universe we live in and not to prove we are in a universe where some event is impossible.

We believe, the natural idea of completeness is different between Mathematics and Physics, because of the discrepancy of the intellectual reality they embody. This is probably due to the fact, that one of the main differences between Mathematics and

[19] Axioms for Mathematics, Laws of nature for Physics, etc.

Physics is that the language of Mathematics is itself the reality Mathematics aims to describe and explain, while the language of Physics is not itself the reality Physics aims to describe and explain. Physics is a language and a science about the **world**. A primary ontology is defined, on which laws of nature act. All of this is made to explain concretely what happens in the world. On the other hand, Mathematics is a language and a science about itself.

For those reasons, metasystematic incompleteness does not imply Physics to be incomplete as a science, but only as a language, while it implies Mathematics to be incomplete as a science, since Mathematics *is* its own language. On one side, Mathematics is a self-referential science and it is the only science with its own language. Mathematics is the most independent and self-referential science. And this is made possible, indeed by the fact that Mathematics language is its own reality. That is why Mathematics does not depend on some external reality and that is why Mathematics is so independent as a science.

On the other side, Physics refers to a concrete and material reality which clearly exists outside the intellectual world. This is not the case for Mathematics, because Mathematics as a science only exists within the intellectual world in the modern sense. We mean, that for a Platonist Mathematics exists just as the computer on which I am writing my essay exists, so it exists by itself. Nonetheless, in the modern sense, which is mainly dominated by formalism, Mathematics exists as a language thanks to the intersubjective reality we create through a common use of our intellect. Mathematics existing out there in the world as Gödel thought, and as every platonist thinks could be empirically complete just as Physics could be empirically complete. Nevertheless Mathematics as being part of the intellectual world in the modern sense, i.e. Mathematics as having only a conceptual existence, can also be empirically complete but only as a system of interrelated concepts and not as a concrete reality.

This is not the only reason, that makes completeness in Mathematics and in Physics different. As it was said above, Mathematics does not aim to understand the world outside itself. Mathematics is the only discipline that has its own unique and comprehensive language. Mathematics as a science and as a language is completely self-sufficient, or at least as self-sufficient as it can be.

Physics has very different goals and intentions. Physics aims at explanation, where Mathematics aims at deduction. As Weinberg says, the goal of Physics is: "the discovery of explanations built into the logical structure of nature".[20] Physics wants to understand how the world works and give the idea of its mechanisms and principles. On the other side, Mathematics wants to deduce exact truth from basic postulates called the axioms.

Neverthless, there is also induction in Mathematics, for example, when a mathematician invents new mathematical tools to solve a given problem and these new tools ends up being whole theories. The invention of imaginary numbers can be mentioned as an example. Nonetheless there are almost no "explanations" in Mathematics, in the sense that Mathematics is not trying to describe what happens in the world and there are almost no deductions in Physics. Physical truths are not deduced from initial postulates, but they are inferred and then validated or not by observations. In Physics nothing is deduced from "axioms", but from observations out of which laws are formulated and out of which

[20] p.10, middle second paragraph, Steven Weinberg, (1992), *Dreams of a Final Theory : The Scientist's Search for the Ultimate Laws of Nature*

explanations are drawn. The same can be said about the physical theory of everything. As it has already been explained several times, the physical theory of everything would not allow someone to deduce perfect and complete knowledge about the whole physical world in its smallest technicalities, but rather to understand the global logical structure of the physical world. In this sense, Physics and also the physical theory of everything is more about *explanations* than *deductions*. Nonetheless, even if Mathematics and Physics are somehow very different, they remain very deeply connected to each other: they both want to establish eternal and perfect truths in their own way and in their own world but with a *shared* language.

It can be said that the completeness of the physical theory of everything is not equivalent to a scientific theory able to completely describe everything that happens in the world. There are different levels of understanding of the reality: physical, biological, chemical, psychologial, etc. The reality is made out of conceptual layers, where each one lies on each other. Physics is the most fundamental layer, but the other layers still explain something of the reality that Physics does not explain, even if Physics was truly complete. Completeness of Physics can be meaningful only about its level of explanation of the reality.

6 The logical compatibility of Gödel's incompleteness theorem with the physical theory of everything and beyond...

In this part, using all the work done throughout the essay, we will examine whether Gödel's incompleteness theorem is logically incompatible with the most complete physical theory of physics: the theory of everything. We will also discuss some further implications GIT probably have for science and knowledge itself.

First, considering what was said above, it becomes clear that GIT only affects metasystematic completeness, while it does not affect empirical completeness at all. Even if we can show that the physical theory of everything respects well enough the conditions of GIT, considering the primitive ontology, the initial conditions of the universe and the laws of nature as axioms for example, GIT could only imply that the physical theory of everything is metasystematically incomplete and not empirically incomplete. From this arises a natural question: "Is it a problem for the physical theory of everything (supposing it exists) and more broadly for Physics to be metasystematically incomplete?"

Naturally, what can express the language of the physical theory of everything goes far beyond the set of potential and actual physical events in the universe. Nonetheless, as it has already being said, the very goal of the physical theory of everything is to give a physical meaning or at least a scientific explanation to all potential and actual physical events in the universe. It is not to give a meaning or a scientific explanation to all the physical events that can be formulated within the language of the physical theory of everything. On a theoretical level, metasystematic incompleteness does not affect the theory of everything and Physics very much, because metasystematic completeness is not a necessary part of what is the physical theory of everything and of what is Physics.

On the practical level, we believe the same holds, i.e. metasystematic incompleteness does not represent a real problem. In fact, it might be said that physical explanations

are just one way to put the world in a certain perspective. The physical perspective of the world truly exists, but in a similar way as language truly exists.[21] The words we are using in this essay exist. Nonetheless if my words or more broadly my language would suddenly cease to exist, the computer on which I am writing this essay would not be affected by this and would simply continue to exist. Naturally, the completeness of the physical world in itself does not depend more on the completeness of Physics, than the existence of this computer depends on the existence of a language to describe it. Therefore, we believe, that we could have a complete physical world with a metasystematically incomplete Physics and even with a necessarily metasystematically incomplete Physics as GIT could imply. For this reason, on a very practical level, the metasystematic incompleteness of the physical theory of everything and of Physics would not affect Physics very much and would let open an infinity of possible researchs within Physics.

Considering what was said, we conclude, that the physical theory of everything and Physics in general are logically compatible with GIT. Indeed, GIT could only imply the physical theory of everything and Physics to be metasystematically incomplete, where the physical theory of everything and in general Physics only aim at empirical completeness. A last question arises from this conclusion: "Can the physical theory of everything be empirically incomplete?". If we suppose such a theory exists or could exist, then by definition it cannot be empirically incomplete. Indeed, the very idea of such a theory is to give an explanation of everything physical that exists or could potentially exist, on a physical level. The latter is equivalent with being empirically complete. So by definition, the physical theory of everything is necessarily empirically complete.

We allow ourselves to have some thoughts beyond the original scope of this essay, but that could nonetheless give a broader meaning to the work done in this essay. GIT had and has enormous consequences for the way we think and see Mathematics. GIT put an end to the biggest dream of Mathematics, that was explicitly stated as one of the main points of Hilbert's program (just one year before GIT was published). The questions of the consequences of such a result of Mathematics for other sciences and for the intellectual world in general naturally arises. Indeed what does or can GIT mean for science and knowledge in general? To answer this question we need to do a little digression within the history of Mathematics and Philosophy.

As it is well explained in Russel's work *A History of western Philosophy*, Mathematics took the place of the ideal of knowledge and even of the paradigm of the intellectual world since Pythagoras. Plato emphasized the importance of Mathematics linking the very idea of an idea to Mathematics. Indeed, an idea according to Plato, had to be perfect, eternal, static, abstract and in its existence had to be completely separated from the material world just as Mathematics is. But this is not everything. The work of linking Mathematics with Philosophy and in a broader sense, with the whole intellectual world, was carried on by Descartes, Pascal, Leibniz, Russel and many others, in many different, but essential ways. The last great intellectual whose work had a great impact for both Mathematics and Philosophy at the same time is Gödel. Gödel arrives at the end of a long history between Mathematics and Philosophy, that deeply shaped the intellectual world.

Indeed, even nowadays, Mathematics remains central in the way we consider knowledge

[21] That would be the "linguistic" perspective of the world or maybe just a perspective of the world, since our experience of the world is from the very beginning linguistic as Gadamer claims.

and science. Science is considered to be science mainly by the use of mathematical tools and by the rigorous use of reasons as it is prescribed by Mathematics. Social sciences, politics, and psychology must use statistics and therefore Mathematics in order to be able to call their conclusions "scientific". The importance of Mathematics for the whole intellectual world and the possible impact that GIT can have for knowledge in itself is confirmed by the words of Carlo Celluci : "Since mathematics has often been regarded as the standard of rational knowledge that other sciences should strive to attain, Gödel's incompleteness theorem seem to acquire significance for the whole body of human knowledge".[22]

GIT implies that truth is not equivalent with being provable. Disregard for our choice of rules, postulates, axioms or whatever we call them, there will always be true statements that are not provable. Therefore, it seems that science, i.e. the set of all the disciplines, that aim at reaching truth by objective proofs empirical or not, could only reach one part of truth, but never truth as a whole. In fact, GIT suggests that there are true statements about the world, that cannot be proved within some formal systems. GIT seems to imply that truth goes beyond what can be proven. Of course, this is only one likely consequence of GIT for knowledge itself. Nonetheless it must be said, that because of the very importance of Mathematics for the whole intellectual world and because of the deep meaning of GIT, these theorems cannot be considered as an isolated mathematical result. GIT necessarily have a significant importance for the whole body of human knowledge, because they force us to reconsider Mathematics as the paradigm of the intellectual world.

7 Conclusion

In this essay we have discussed the logical compatibility of Gödel's incompleteness theorem with the physical theory of everything and with Physics in general. First, we have explained what GIT is, how it was proven, what is its logical and mathematical meaning and which kind of impact it had for Mathematics as a whole. Second, we have established what would be a physical theory of everything, supposing it would exist. We have explained in which sense, it would be a fundamental theory for Physics, and which place it would have within science. Third, since Gödel's incompleteness theorems are about the logic of formal systems, we considered in which sense Mathematics are and can be formal systems. Indeed, this is essential to a possible application of Gödel's incompleteness theorems, since GIT can only be applied to very specific formal systems. Fourth, we discussed the idea of completeness in Mathematics and Physics and established that empirical completeness is the most meaningful idea of completeness for the physical theory of everything and for Physics in general. Finally, we concluded that Gödel's incompleteness theorems are definitely compatible with the idea of a complete fundamental physical theory, that we called the physical theory of everything. As an extra comment, we allowed ourselves to consider the themes presented in the essay from a larger perspective, in order to better understand the meaning of GIT and of our work in general. We expounded briefly the importance of Mathematics and therefore the importance of GIT for the whole body of rational knowledge. This topic could be explored further for some further researches.

This philosophical reflection, we have done throughout the redaction of this essay allowed us to have a broader and more precise understanding of the subjects we study:

[22] page 103 of *Gödel's incompleteness theorem and the philosophy of open systems*, Carlo Cellucci

Mathematics and Physics.[23] The similarities and differences of both subjects do appear now in a clearer way to us than before completing this work. We realized that even if Mathematics and Physics share a common language and a common reality, they have very different goals. Their nature as discipline is very different, and some concepts as completeness for example, do not mean the same thing in Mathematics as in Physics. Indeed, there are many ideas, that are very different between Mathematics and Physics. This makes us wonder, how such a great collaboration between Mathematics and Physics was made possible throughout history. This work also motivated us to better understand the subjects we study from both scientific and philosophical points of view.

The writing of this essay, brought into light other types of questions more existential, but still deeply connected with Mathematics and logic. As discussed in the essay, Mathematics is usually considered as the very idea of reason. GIT implies that Mathematics cannot be its own basis, or cannot justify itself. This means that Mathematics does not suffice to itself. We think this implies, that reason as well cannot be its own basis and cannot justify itself. Human reason does not suffice to himself. This might imply that something outside the sphere of rationality gives and must give meaning to reason itself. This would be the failure of rationalism, as the belief to make reason the beginning and the end of everything.

Considering Gödel's incompleteness theorem from a more existential perspective makes us think that reason and more generally intelligence do not exist by themselves, as a light enlightening the world, the reality, and ourselves. Intelligence exists from the beginning as a relation between the world and ourselves or as a relation between ourselves. This leads us to formulate an old truth, forgotten by the technical world in which we find ourselves, that maybe it is the world that gives sense to our intelligence and not our intelligence that gives meaning to the world as we, heirs of the rationalism of the Enlightenment, usually tend to think.

[23] Paul and me study applied Mathematics and Tom studies theoretical Physics.

References

Webography

- https://plato.stanford.edu/entries/goedel-incompleteness/ (viewed the 17.11.2021)

Bibliography

Primary sources:

- Duparc, J. (EPFL course fall 2020), "Gödel and recursivity"
- Steven Weinberg, (1992), *Dreams of a Final Theory : The Scientist's Search for the Ultimate Laws of Nature*
- Cellucci, C. (1992), Gödel's incompleteness theorem and the philosophy of open systems, *Travaux de logique*, 7, p.103-127

Secondary sources:

- Agazzi, E. (1992), Some philosophical implications of Gödel's theorem, *Travaux de logique*, 7, p.129-159
- Barrow J. D. (2011), Gödel and Physics, *Kurt Gödel and the foundations of mathematics*, 11, p.255-276
- Spinoza B. , (1677), Ethics, chap 1-2
- Esfeld M., (EPFL course fall 2021), "Philosophical perspectives on the exact sciences", slides 22 sept.